Piscataway High School
AP Calculus BC
Summer Assignment 2023

To help you prepare for AP Calculus BC, you are required to complete this assignment. These problems will allow you to review and master the prerequisite skills necessary for this rigorous course so that you will be fully prepared in September. The assignment will be graded based on effort and accuracy. I recommend you read Chapter 1 and take notes on vocabulary and theorems. Please expect a test on Chapter 1 during the first week of school. This assignment is due the first day of class in September. For Part 1, you will be asked to write the answers to a few randomly selected problems and part 2 will be collected.

Part 1

In a neat and organized manner, work must be shown to support your answer. Please put work and answers on separate sheets of paper and circle or highlight final answer. An attempt must be made to answer ALL problems. Your textbook and information available online are great resources if you need help. You will need to use your graphing calculator on the last page of this packet. All other pages should be completed without the aid of a graphing calculator, but you may utilize your calculator to check your answers.

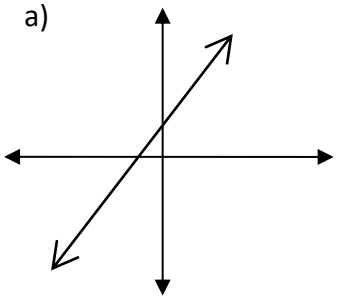
Part 2

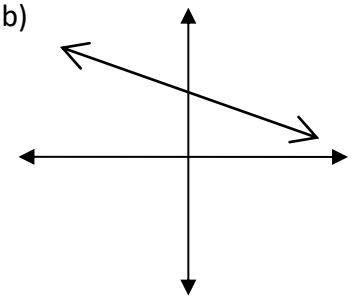
Work must be shown to support your answer. An attempt must be made to answer ALL problems. Note that the problems with a grey background do not permit the use of a calculator.

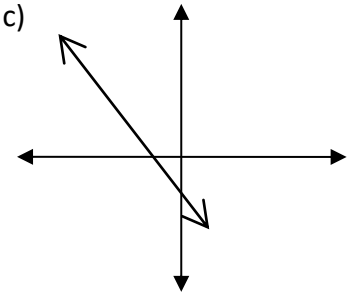
Section	Page(s)	Problems to Complete
Section 1 Exercises	Pages 9–11	#s 1, 5, 12, 13, 16, 23, 29, 32, 35, 37, 41, 47, 48, 49, 54, 55, 57
Section 2 Exercises	Pages 19–21	#s 1-3, 5, 7, 8, 9, 10, 15–31 (odd), 32, 33, 35, 39, 41, 45, 51, 58, 59, 62, 67, 69. Write the domain and range in interval notation.
Section 3 Exercises	Pages 26–28	#s 1-3, 5, 7, 13-18, 21, 23, 24, 27, 32–36, 42, 48. No calculator/Desmos permitted for #s 1-3
Quick Quiz	Page 28	#s 1–4
Section 4 Exercises	Pages 33–35	#s 4, 5-7, 10, 12, 17, 22, 37–42
Section 5 Exercises	Pages 43–44	#s 3–24 (multiples of 3), 33, 36-39, 42, 43, 48, 53–57
Section 6 Exercises	Pages 51–53	#s 1, 5, 6, 9, 13, 18, 21, 27–29, 42, & 50–55
Quick Quiz	Page 54	#s 1–4

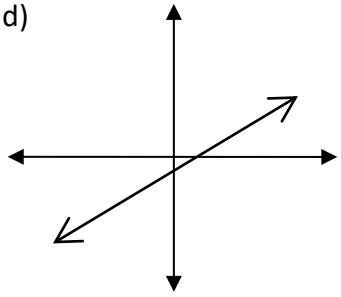
Part 1-Prerequisite Review Problems

1. List the lines in parts a) – d) in order of increasing slope.

a) 

b) 

c) 

d) 

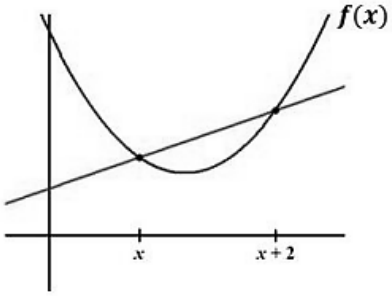
2. Derive Point-Slope Form.

3. Find the Point-slope form of the line which satisfies the following given conditions:

a) The line parallel to the line $y = 4x - 2$ and passes through the point $(3, -4)$.

b) The line perpendicular to the line $x - 4y = 7$ and passes through the point $(3, -4)$.

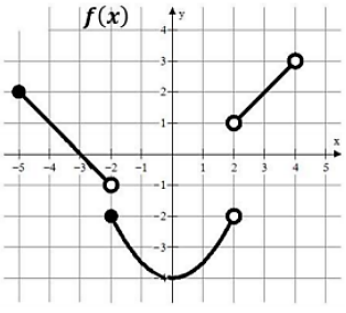
4. Write the slope of the line given below using function notation.



5. Find the points of intersection for $y = x^2$ and $y - 2 = x$ algebraically.

6. Find the points of intersection for $y = e^x$ and $y = x + 1$ graphically.

7. Refer to the graph given on the right on the interval $[-1, 2]$.



a) Does the graph represent a function?
Explain your answer.

b) Find $f(-2)$, if it exists.

c) Find $f(f(0))$, if it exists.

8. Find the equation of the circle in standard form satisfying the given condition:

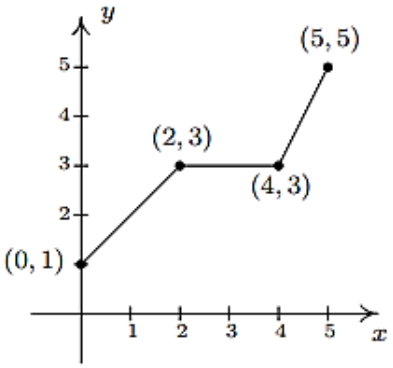
a) Center (3, –2), radius = 4

b) Center (4, –5), passes through (1, 3)

9. Use the graph of $f(x)$ given, to sketch the following 5 transformations. Do not graph all on the same coordinate plane.

a. $h(x+1)$ b. $3h(x)$ c. $1-h(x)$

d. $-h(x)$ e. $h(-2x)$

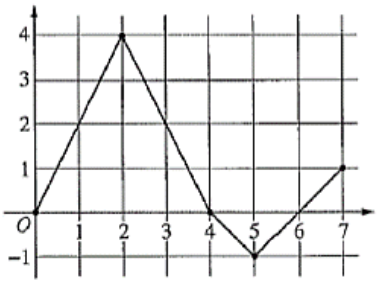


$y = f(x)$

10. The function g is a transformation of f and the graph f is given below.

a. Write an equation of g in terms of f such that the minimum value of g is located at the point $(2, -4)$.

b. Write an equation of g in terms of f such that the range of g is $[0,4]$.



Graph of f

For numbers 11-18, find the domain of the function algebraically. Write your answer in interval notation

11. $y = x^3 - 3x^2 + 3x - 2$

12. $y = \sqrt{x^2} - 1$

13. $y = \dfrac{x}{x^2+1}$

14. $y = \dfrac{1}{\sqrt[3]{1-x^2}}$

15. $y = ln(x-5)$

16. $y = \frac{1}{\sqrt{x^2-2}}$

17. $y = xe^{-x}$

18. $y = x^{3/2}$

For numbers 19-30, graph the following functions, plotting one or two critical points and any asymptotes. You should be able to quickly sketch the graphs without making an x-y table. Finding the domain, range, end behavior, and intercepts and recognizing any transformations will be helpful.

19. $y = \frac{1}{x-3} - 1$

20. $y = \frac{x^2 - x - 6}{x - 3}$

21. $y = \sqrt{x^2}$

22. $y = \sqrt{x + 5}$

23. $y = \sqrt{25 - x^2}$

24. $y = 1 + \sqrt{4 - x}$

25. $x = -\sqrt{4 - y^2}$

26. $y = e^{-x}$

27. $x = y^2 + 2$

28. $y = \sqrt[3]{x}$

29. $y = |x^3|$

30. $y = \ln|x|$

31. Sketch the curve defined by the equation given below by finding two functions y_1 and y_2 whose graphs will combine to give the curve, if possible.

a. $x - y^2 = 0$ b) $x^2 + y^2 = 9$ c) $x^3 + y^3 - 9xy = 0$

32. For the following questions, find the end behavior by filling in the blanks.

a) Let $f(x) = x^2 - 7x + 4$.

If $x \to \infty$ then $f(x) \to$ _____
If $x \to -\infty$ then $f(x) \to$ _____

b) Let $g(x) = \frac{1}{x}$

If $x \to \infty$ then $g(x) \to$ _____
If $x \to -\infty$ then $g(x) \to$ _____

33. Find the end behavior of $g(x) = 2^x$ without relying on the graph. Explain your answer.

If $x \to \infty$ then $g(x) \to$ _____
If $x \to -\infty$ then $g(x) \to$ _____

34. Which function grows faster as $x \to \infty$?

a) $f(x) = x^{20}$ or $g(x) = 4^x$ b) $f(x) = ln(x)$ or $g(x) = x$

c) $f(x) = 2^x$ or $g(x) = 4^x$ d) $h(x) = sin(x)$ or $g(x) = ln(x)$

35. Let $f(x) = 2x^3 - 3x - 7$. Find the following.

a. $f(-2)$ b. $f\left(-\frac{1}{2}\right)$ c. $f(x+h)$

36. If $f(x) = x^2 + 2x - 5$, find $\frac{f(x+h) - f(x)}{h}$ and simplify completely.

37. If $f(x) = x^2 - 1$ and $g(x) = 3x + 1$, find $f\left(g(x)\right)$.

For numbers 38-40, determine functions f and g such that $h(x) = f(g(x))$. (Do not choose $f(x) = x$ or $g(x) = x$).

38. $h(x) = \sqrt{x^2 - 4}$

39. $h(x) = e^{-4x}$

40. $h(x) = sin^3 x$

41. Expand the expression given below.

$3(x + 2y)^2 - (2x + y)^2$

42. Solve for y': $\frac{y - xy'}{y^2} = \frac{xy' - y}{x^2}$

43. Solve for c: $a^2c - 2ab = 4a - b$

For numbers 44-51, simplify the following problems without writing every intermediate step.

44. $\frac{7}{3a - b} + \frac{5}{2b - 6a}$

45. $\frac{\frac{1}{4+h} - \frac{1}{4}}{h}$

46. $2y - 3 - \frac{3y}{y + 3}$

47. $\frac{1}{\sqrt{2 - \frac{1}{x^2}}}$

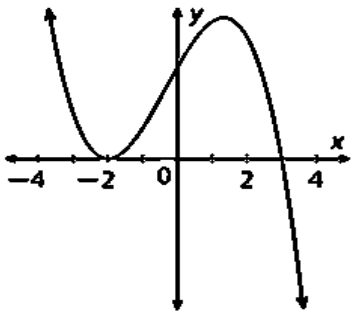
48. $\left(\frac{a-b}{2ab}\right) \div \left(\frac{1}{b^2} - \frac{1}{a^2}\right)$

49. $\left(\frac{-2x^2}{y^3}\right)^3 \left(\frac{x^{-1}}{3y^{-3}}\right)^2$

50. $\left(1 + x^{-1}\right)^{-1}$

51. $\sqrt{\frac{12x^3y^5}{x^2}}$

For numbers 52-54, use the graph below to answer the following questions. Write your answer in interval notation where needed.

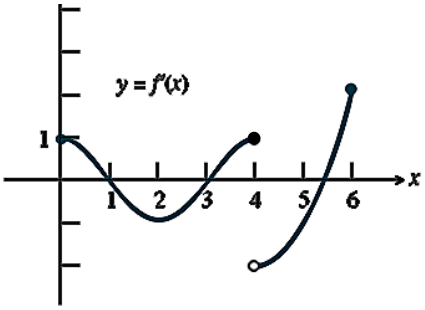


52. For which interval(s) is the function decreasing?

53. For which *x*-value(s) does the function change signs?

54. On which interval(s) is the function positive?

55. Use the graph given to state the *x*-value(s) where the function changes signs.



For numbers 56-58, use algebraic methods by creating a sign chart to state the x-value(s) where the function changes signs.

56. $f(x) = x^2 - 4$

57. $f(x) = xcos(2x)$ on $[0, \pi]$

58. $f(x) = \frac{x^2-2}{x^2-4}$

For numbers 59-61, solve the non-linear inequalities graphically. Write your answer in interval notation.

59. $x(x-3)(x+2)^2 > 0$

60. $\ln(x) < 0$

61. $\sqrt{16-x^2} > 0$

For numbers 62-64, solve the non-linear inequalities algebraically by using a sign chart. Write your answer using interval notation.

62. $2x^2 - x - 3 < 0$

63. $e^x + xe^x > 0$

64. $\cos\left(\frac{\pi}{6}x\right) > 0$ on $[0,12]$

65. Express the area of a square ***A*** as a function of its diagonal, ***d***.

66. Express the area of an equilateral triangle ***A*** as a function of its side length, **s.**

67. The following is a multiple choice question. Select the best answer.

Let f be the function defined by $f(x) = 2x^3 - x$. Which of the following expressions is the average rate of change of f on the interval $[1,3]$?

A. $\frac{f(3)+f(1)}{2}$

B. $\frac{f(1)-f(3)}{3-1}$

C. $\frac{f(3)-f(1)}{3-1}$

D. $f(3) - f(1)$

68. If a ball is thrown straight up with an initial velocity of 32 feet per second, then after **t** seconds, the distance **s** above its starting height, in feet, is given by $s = 32t - 16t^2$. Use the graph of $s(t)$ to find what time the ball will reach its highest point and how high will it rise.

69. A rectangular field is to be enclosed with 500 ft. of fencing along three sides and by a straight stream on the fourth side. Let x be the length of each side perpendicular to the stream, and let **y** be the length of the side parallel to the stream.

a) Draw a figure representing the situation modeled above.

b) Express y in terms of x.

c) Find a formula for the area A of the field in terms of x.

d) What is the largest area that can be enclosed?

For numbers 70 and 71, use the given the conditional to write its converse and contrapositive. Then decide whether the three statements are true or false.

70. If a figure is square than it is a rectangle.

71. If a number is divisible by 3 then it is divisible by 6.

72. Use the previous problem to explain the relationship between the conditional and contrapositive.

For numbers 73-75, the angle θ is an acute angle of a right triangle. Solve the following problems by drawing an appropriate right triangle.

73. Find $\sin\theta$ and $\tan\theta$, given that the hypotenuse has length 5 and the side adjacent to θ has length 2.

74. Find $\tan\theta$ and $\csc\theta$, given $\sec\theta = \frac{5}{3}$.

75. Find the length of the side opposite θ, given that the side adjacent to θ has length 15.6 and $\cot\theta = 5.2$

For numbers 76-79, find all values of θ in the interval $(0, 2\pi]$ that satisfy the equation.

76. $sin\ \theta = \frac{-1}{2}$

77. $csc\ \theta = \frac{2}{\sqrt{3}}$

78. $cot\ \theta = -1$

79. $sin\ (2\theta) = -1$

For numbers 80-82, determine if the equations are valid for all θ? Reminder, show work.

80. $sin\ (\theta + 2\pi) = sin\ \theta$

81. $cos\ (-\theta) = cos\ \theta$

82. $sin\ (\pi + \theta) = -sin\ \theta$

For numbers 83-85, evaluate the expression given.

83. $arcsin\ 0$

84. $cos\left(arcsin\ \frac{3}{5} + arctan\ \frac{5}{12}\right)$

85. $cos\ (arctan\ 2\ )$

For numbers 86-89, sketch the following graphs and give the amplitude and period of each.

86. $y = cos\left(\frac{1}{3}x\right)$

87. $y = |sin(x)|$

88. $y = 2 - cos\ x$

89. $y = 2\ sin\left(4x + \frac{\pi}{4}\right)$

90. Use the properties of logarithms to rewrite $\log_a\left(\frac{x^{1/2}\,y^2}{z^{3/2}}\right)$ in expanded form in terms of x, y, and z.

For numbers 91 and 92, determine if the following are equivalent statements. If they are not, explain why or rewrite one side to make equivalent.

91. $log(m+n) = log(m) \cdot log(n)$

92. $(log_b x)^3 = 3log_b x$

For numbers 93-97, simply the expression.

93. $ln\left(e^{tan\theta}\right)$

94. $log_{25}\left(\frac{1}{5}\right)$

95. $\ln\left(3e^{7r^2-13}\right)$

96. $e^{2x+\ln 5}$

97. $log_2\left(\frac{2^{x+3}}{2^{x-1}}\right)$

For numbers 98 and 99, graph the function given, showing at least 2 points and any asymptotes.

98. $y = \log_5 x$

99. $y = 2(3^x)$

For numbers 100-108, solve algebraically. Justify your answer graphically.

100. $2x^{-1/3} - 1 = 0$

101. $\frac{2x^2 - 3x}{\sqrt{x+5}} = 0$

102. $2e^{5x} = 5$

103. $3^x = \frac{1}{81}$

104. $4e^{4x} + 16xe^{4x} = 0$

105. $\log(3x + 5) = 1$

106. $\log(2x + 1) - \log(3x - 4) = 1$

107. $\left|\frac{x - 3}{x + 4}\right| = 5$

108. $|3x| \leq |2x - 5|$

Ti-89 Graphing Calculator Skills

1. Mode button

   - Know how to change from degrees to radians
   - Know the difference between exact vs auto vs approx.
   - Know how to convert answer from exact to approx.

2. Graphing functions

   - Practice graphing functions: #s 20, 28, 29, 89
   - Can you graph # 27 using your calculator?
   - Make sure you know how to change the windows so you can see the entire graph: # 69 part d
   - Make sure you know how to create an $x - y$ table: # 34

3. Know how to find points of intersection graphically and algebraically: #s 5 and 6

4. Know how to solve an equation graphically and algebraically: # 106

5. Know how to solve a trigonometric equation: # 79

6. Know how to solve a nonlinear inequality: # 63 and 64